Design for Lifecycle Cost using Time-Dependent Reliability Analysis

Zissimos P. Mourelatos
Amandeep Singh
Jing Li

Mechanical Engineering Department
Oakland University
Rochester, MI 48309, USA
mourelat@oakland.edu

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **04 FEB 2008**	2. REPORT TYPE **Briefing Charts**	3. DATES COVERED **04-02-2008 to 04-02-2008**

4. TITLE AND SUBTITLE **DESIGN FOR LIFECYCLE COST USING TIME-DEPENDENT RELIABILITY ANALYSIS**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) **Amandeep Singh; Zissimos Mourelatos; Jing Li**	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Oakland University,Mechanical Engineering Department,Rochester,MI,48309**	8. PERFORMING ORGANIZATION REPORT NUMBER **; #22497**
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **U.S. Army TARDEC, 6501 E.11 Mile Rd, Warren, MI, 48397-5000**	10. SPONSOR/MONITOR’S ACRONYM(S) **TARDEC** 11. SPONSOR/MONITOR’S REPORT NUMBER(S) **#22497**

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

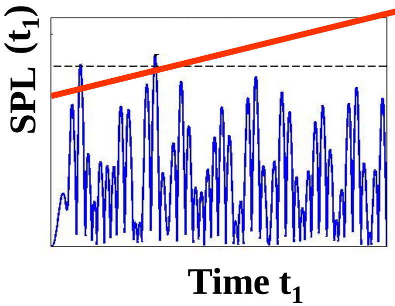
13. SUPPLEMENTARY NOTES

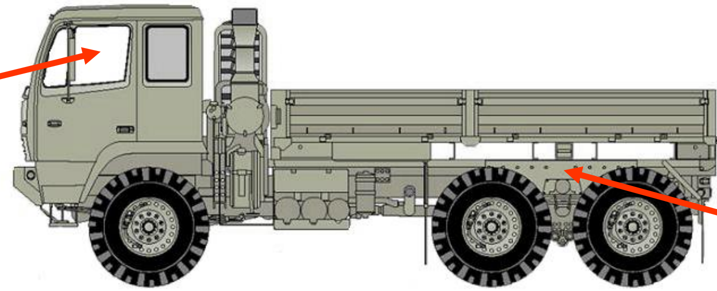
14. ABSTRACT
N/A

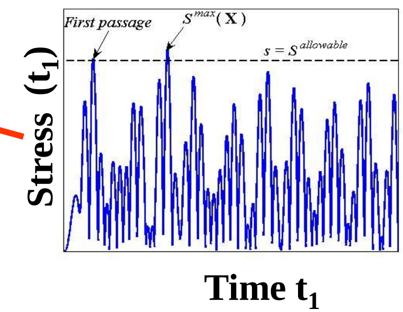
15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**33**	

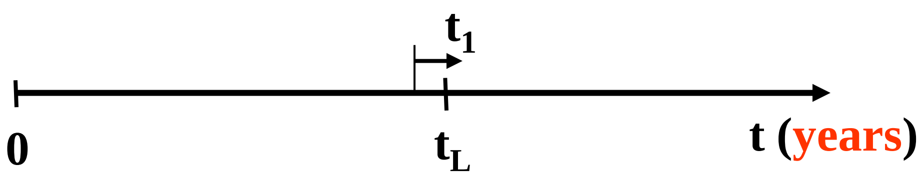
Problem Definition







Response(t) = f [E(t), Degradation/Wear(t), Load(t_1)]



$$E(t) = E_0(1 - D * t) \longrightarrow E(t_L) \text{ is a R. V.}$$

Random Process

Degradation Rate (Random Variable)

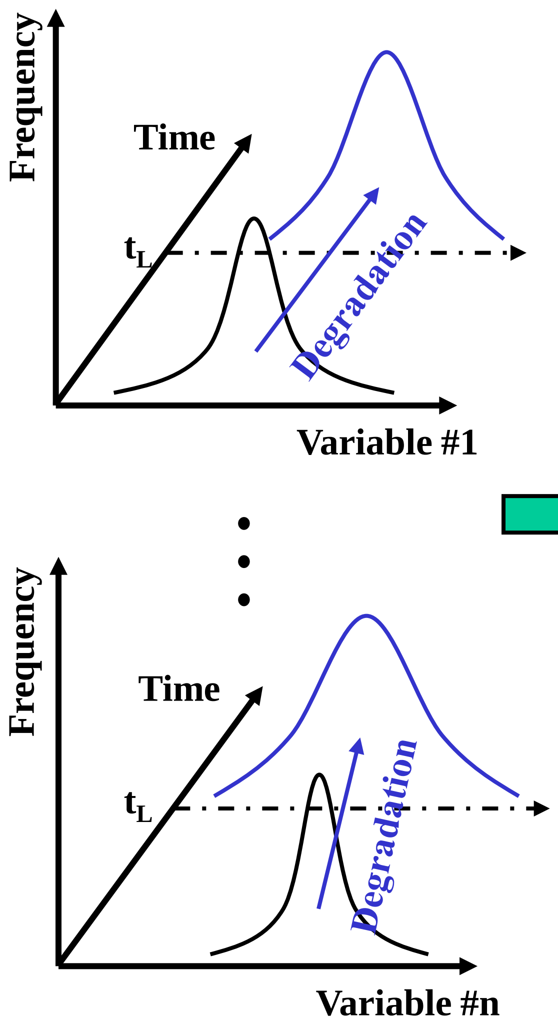
Two time scales; t and t_1

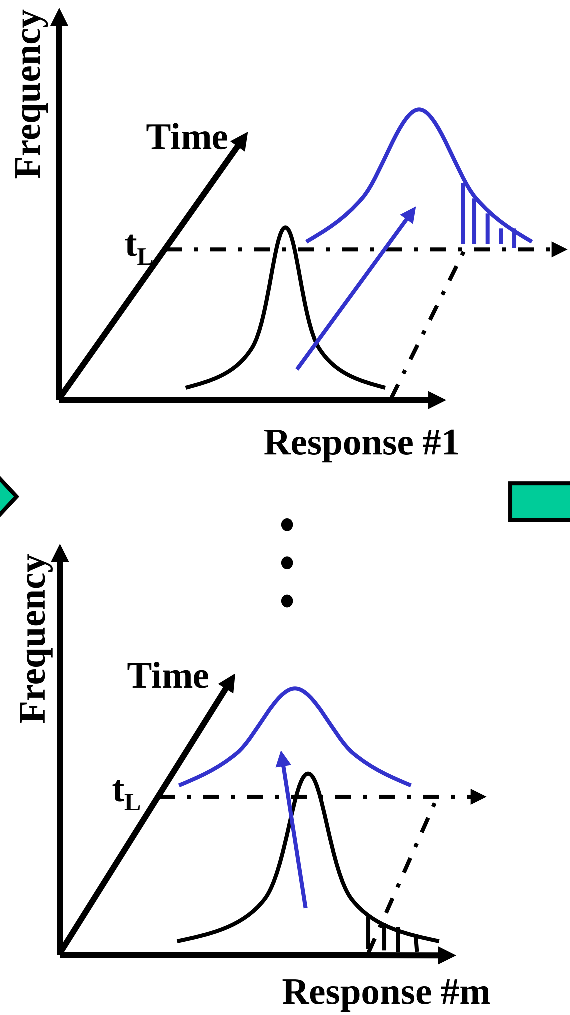
Problem Definition

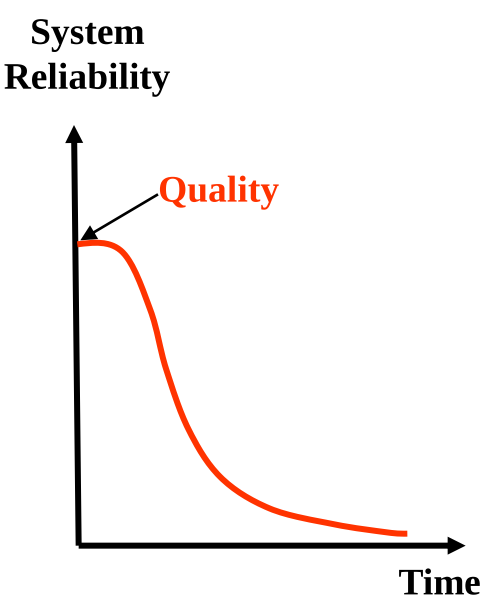
Input variables

System Responses

System Reliability







Quality = Reliability (t = 0)

Definitions / Observations

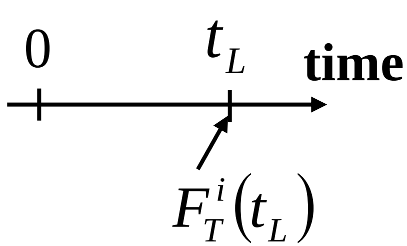
Reliability: Ability of a system to carry out a function in a time period [0, t_L]

$$p_f^c = P(t \le t_L) = F_T^c(t_L)$$ **Prob. of Time to Failure**

$$\boxed{F_T^c(t_L) = P(\exists t \in [0, t_L], such \quad that \quad g(\mathbf{X}(t), t) \le 0)}$$ **Cumulative Prob. of Failure**

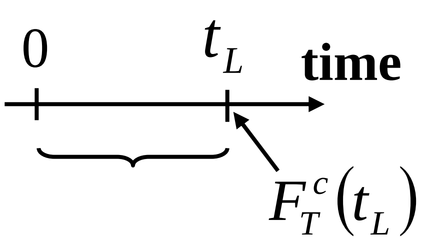
$$\boxed{F_T^i(t_L) = P(g(\mathbf{X}(t_L), t_L) \le 0)}$$ **Instantaneous Prob. of Failure**

Time-Invariant Reliability



$\neq$

Time-Variant Reliability



Definitions / Observations

Reliability

Mechanical (or Structural) Reliability

(“Hard” Failures)

Performance Reliability

(“Soft” Failures)

Quality: Performance Reliability at t = 0

OR

Conformance to Specifications at t = 0

Performance Reliability = Quality over Time

Design for Lifecycle Cost

$$C_L\left(\mathbf{d},\mathbf{X},t_f,r\right)=C_P\left(\mathbf{d},\mathbf{X}\right)+C_I\left(\mathbf{d},\mathbf{X},t_0\right)+C_V^E\left(\mathbf{d},\mathbf{X},t_f,r\right)$$

Lifecycle Cost

Production Cost

Inspection Cost

Expected Variable Cost

Final time

Interest rate

$$C_V^E\left(\mathbf{d},\mathbf{X},t_f,r\right)=\int_0^{t_f}c_F(t)e^{-rt}f_T^c(t)dt$$

Cost of failure at time t

PDF of time to failure

$$F_T^c(t_L)=P\left(\exists t\in[0,t_L],\, such \quad that \quad g\left(\mathbf{X}(t),t\right)\le 0\right)$$

Design for Lifecycle Cost

# of time-dependent limit states

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}}} C_L\left(\mathbf{d},\mathbf{X},t_f,r\right)$$

s. t. $F_{T_i}^{c}\left(\mathbf{d},\mathbf{X},t\right)\le p_{f_i}^{t}$ **for** $i=1,2,\cdots,n$

$$\mathbf{d}_L\le\mathbf{d}\le\mathbf{d}_U$$

$$\boldsymbol{\mu}_{\mathbf{X}_L}\le\boldsymbol{\mu}_{\mathbf{X}}\le\boldsymbol{\mu}_{\mathbf{X}_U}$$

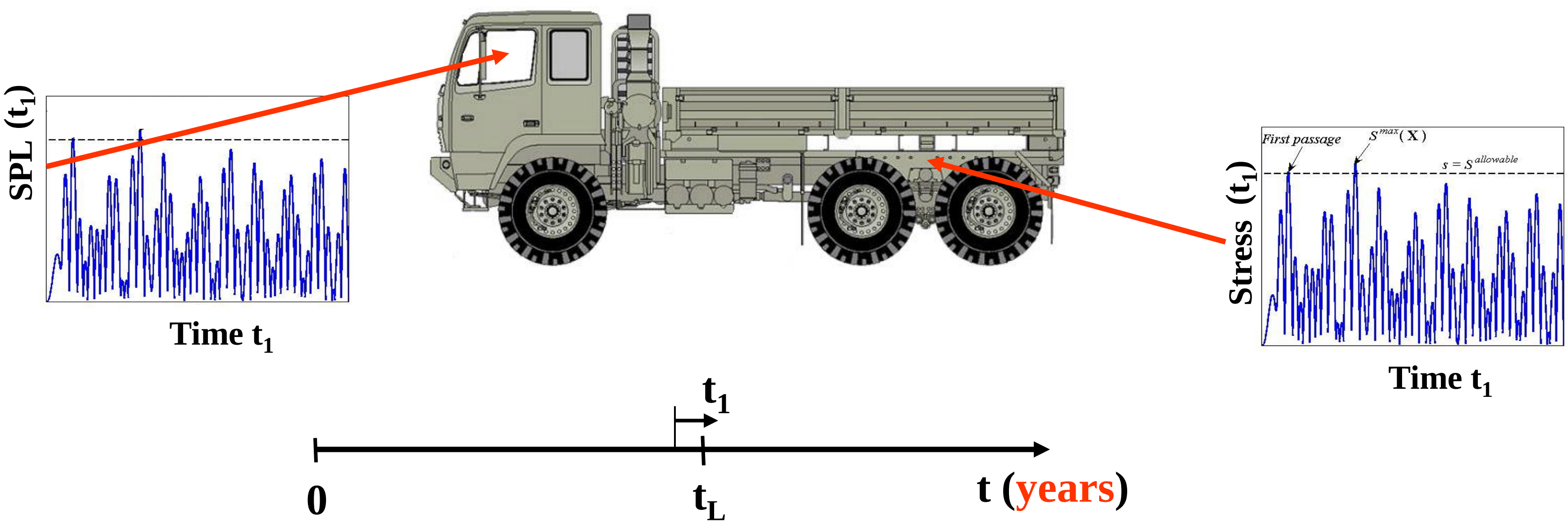
$$\boldsymbol{\sigma}_{\mathbf{X}_L}\le\boldsymbol{\sigma}_{\mathbf{X}}\le\boldsymbol{\sigma}_{\mathbf{X}_U}$$

e.g. $p_f^t=0.05$ **after** t **= 5 years**

Calculation of Cumulative Probability of Failure

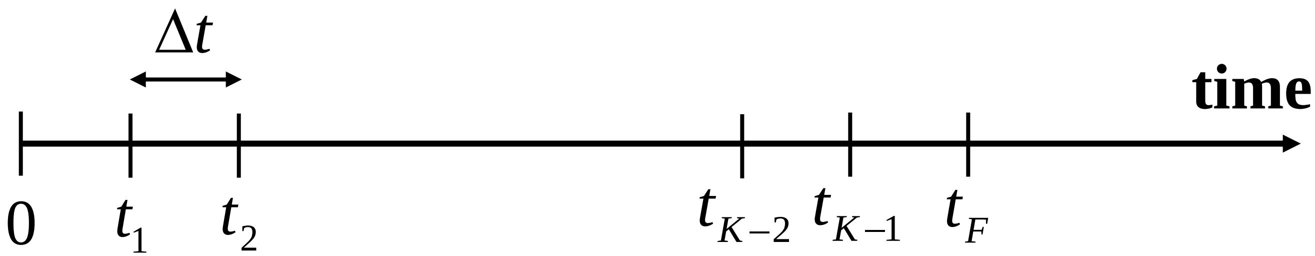
- Series system approach for [0, t_f]
- Reliability estimation at time t_L ($0 < t_L < t_f$);

multiple MPP case



Calculation of Cumulative Probability of Failure

$$F_T^c(t_F) = P\left(\exists t \in [0, t_F], \textit{such} \quad \textit{that} \quad g(\mathbf{X}(t), t) \le 0\right)$$



$$t_F = K\Delta t$$

$$\boxed{F_T^c(t_F) = P\left(\bigcup_{k=0}^{K} g(\mathbf{X}(t_k), t_k)\right)}$$

Series System
Prob. of Failure

Cumulative Probability of Failure (PHI2 Method)

$$\boxed{F_T^c(t_F) \approx F_T^i(0) + E\left[N^+(0,t_F)\right]}$$

up-crossing rate

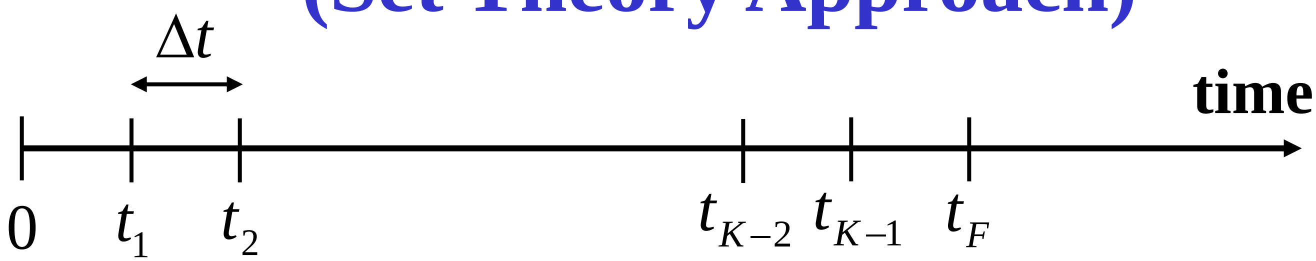
$$E\left[N^+(0,t_F)\right] = \int_0^{t_F} \nu^+(t)\,dt$$

# of up-crossings in $\left[0, t_F\right]$

$$\nu^+(t) = \lim_{\Delta t \to 0} \frac{P\left[\left(g\left(\mathbf{X}(t),t\right) > 0\right) \cap \left(g\left(\mathbf{X}(t+\Delta t), t+\Delta t\right) \le 0\right)\right]}{\Delta t}$$

$$\nu^+_{PHI\,2}(t) = \frac{\Phi_2\left[\beta(t), -\beta(t+\Delta t), \rho_{gg}(t, t+\Delta t)\right]}{\Delta t}$$

Cumulative Probability of Failure (Set Theory Approach)



$$t_F = K\Delta t$$

$$F_T^c(t_F) = P\left(\bigcup_{k=0}^{K} g\left(\mathbf{X}(t_k), t_k\right)\right)$$

Define: $IF_k = \left\{g\left(\mathbf{X}(t_k), t_k\right) \le 0\right\}$ **(Instantaneous Failure Event)**

then: $\mathbf{CF}_K = \bigcup_{k=0}^{K} IF_k$ **(Cumulative Failure)**

$\mathbf{\Delta F}_k = \overline{\mathbf{CF}_k} \cap \mathbf{CF}_{k+1}$ **(Incremental Failure)**

$$\boxed{P(\mathbf{\Delta F}_k) \approx P(\mathbf{CF}_k \cup \mathbf{CF}_{k+1}) - P(\mathbf{CF}_k)}$$

Cumulative Probability of Failure (Set Theory Approach)

$$\boxed{F_T^c(t_F) = P\left(\bigcup_{k=0}^{K} g\left(\mathbf{X}(t_k), t_k\right)\right) \approx P(IF_0) + \sum_{k=0}^{K-1} P(\Delta \mathbf{F}_k)}$$

$$P(\Delta \mathbf{F}_k) \approx P(\mathbf{CF}_k \cup \mathbf{CF}_{k+1}) - P(\mathbf{CF}_k)$$

Probabilistic Re-Analysis MC Method

Cumulative Probability of Failure (PHI2 Method)

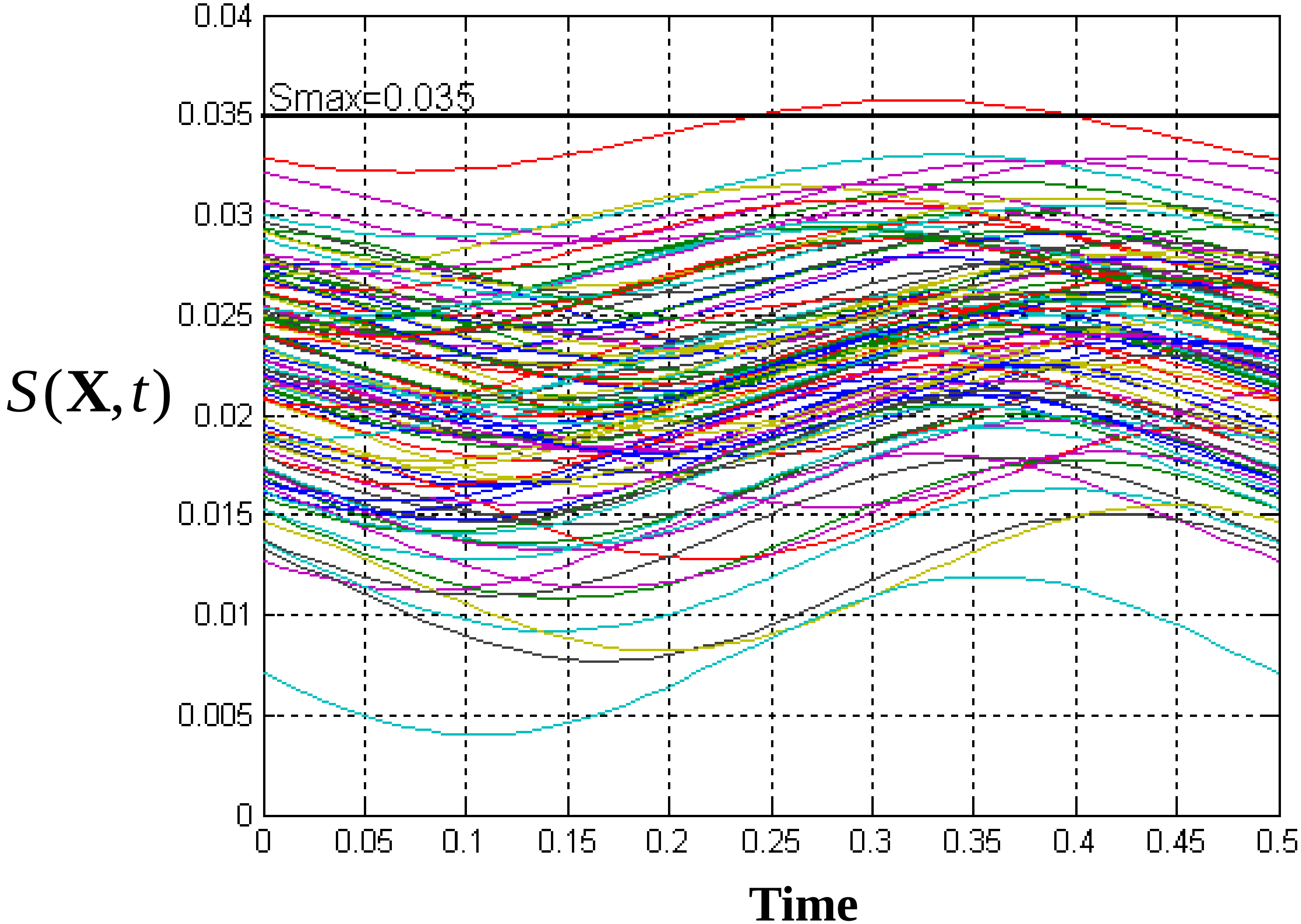
$$\boxed{S(\mathbf{X},t) = \frac{1}{18} - \left[12{,}000 - 1{,}000\sin\left(4\pi t + X_3\right)\right] * \frac{10^4}{X_1 X_2}}$$

$$X_1 \sim N\left(2.29 * 10^{-3}, 0.229 * 10^{-3}\right)$$

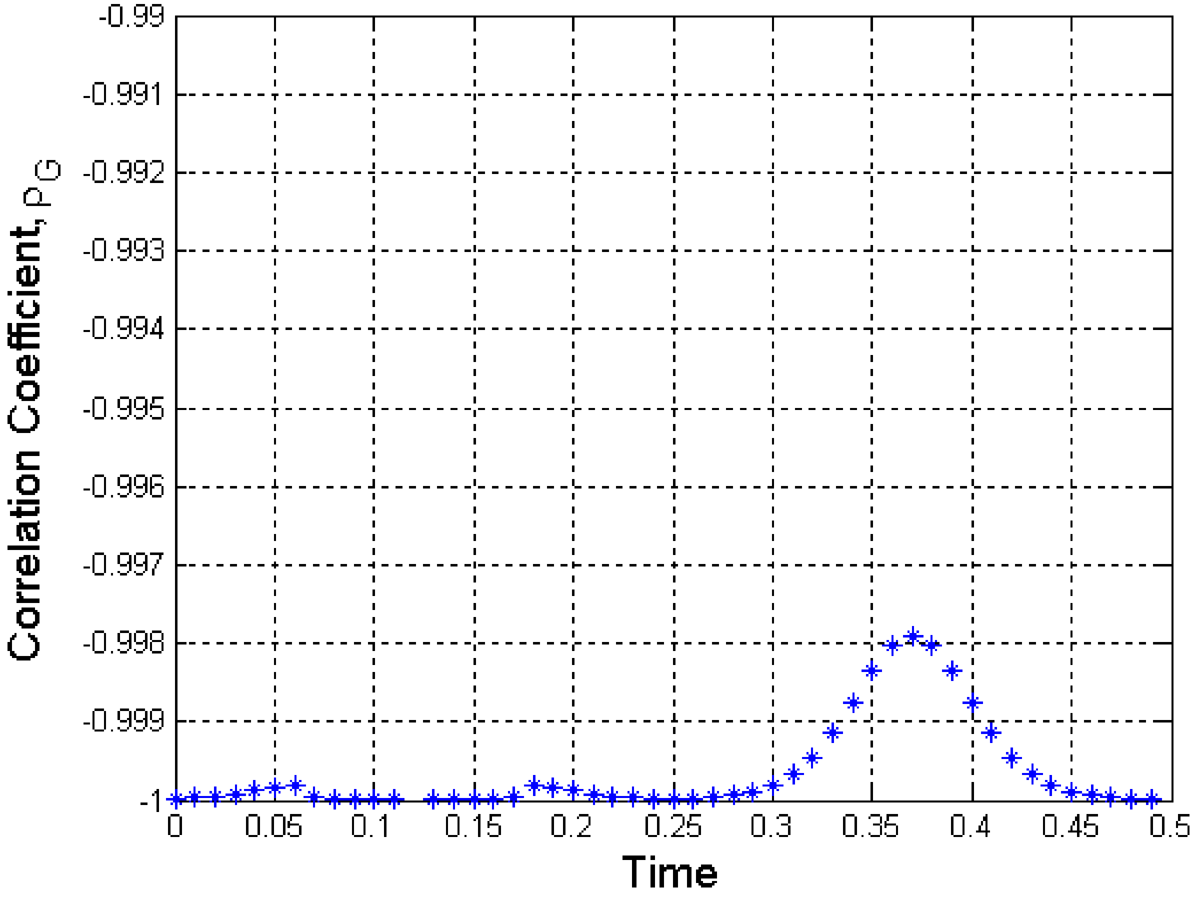
$$X_2 \sim N\left(2 * 10^{11}, 0.2 * 10^{11}\right)$$

$$X_3 \sim N\left(0, 0.7\right)$$

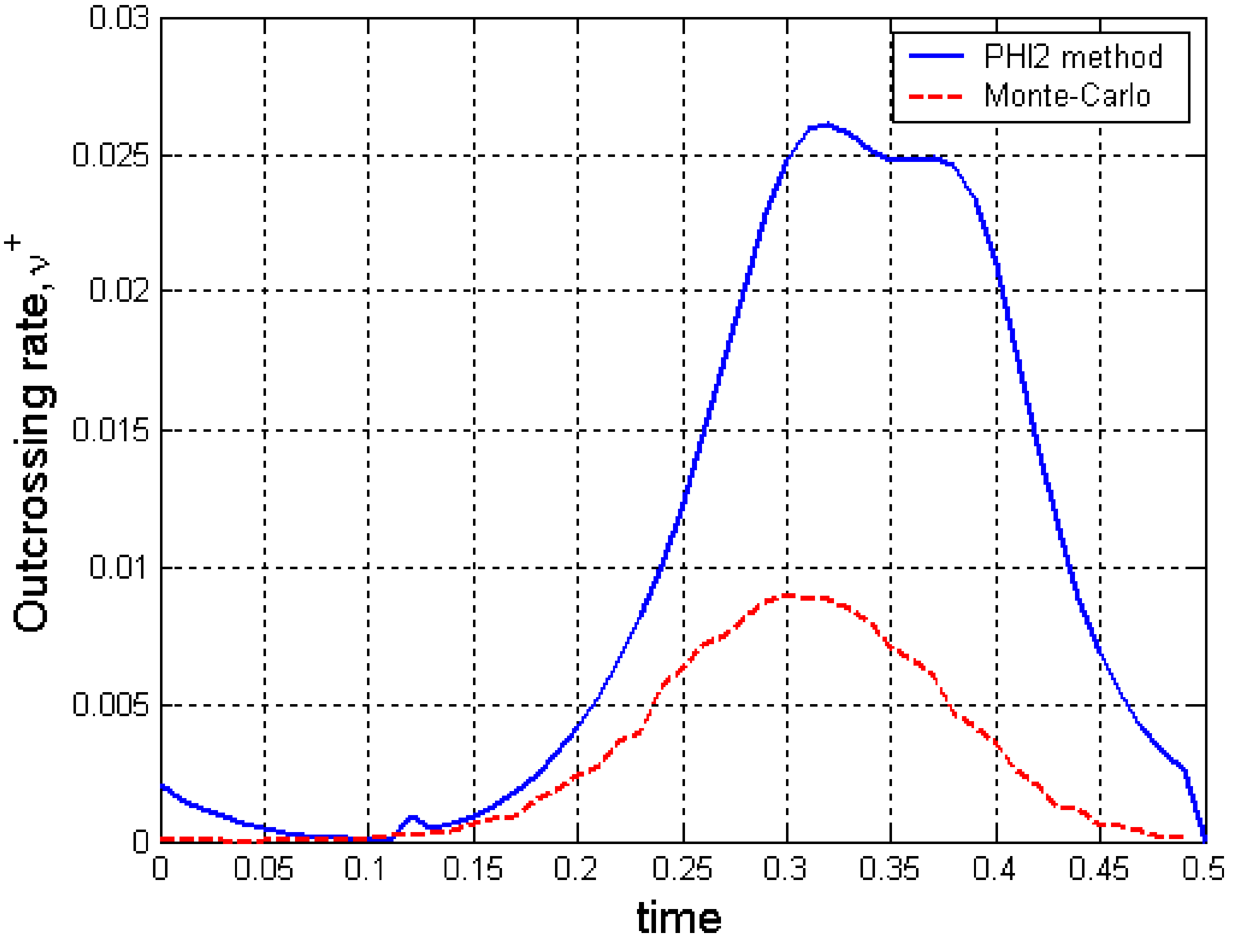
Cumulative Probability of Failure (PHI2 Method)



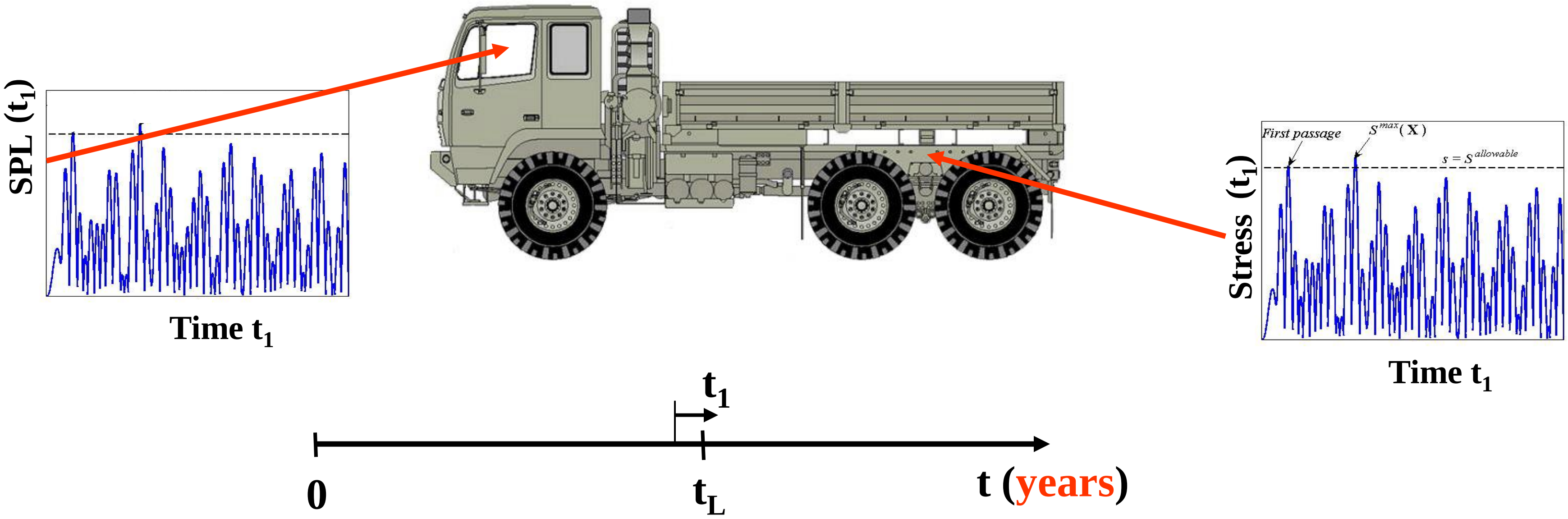
Cumulative Probability of Failure (PHI2 Method)



Cumulative Probability of Failure (PHI2 Method)



Reliability Estimation at Time t_L



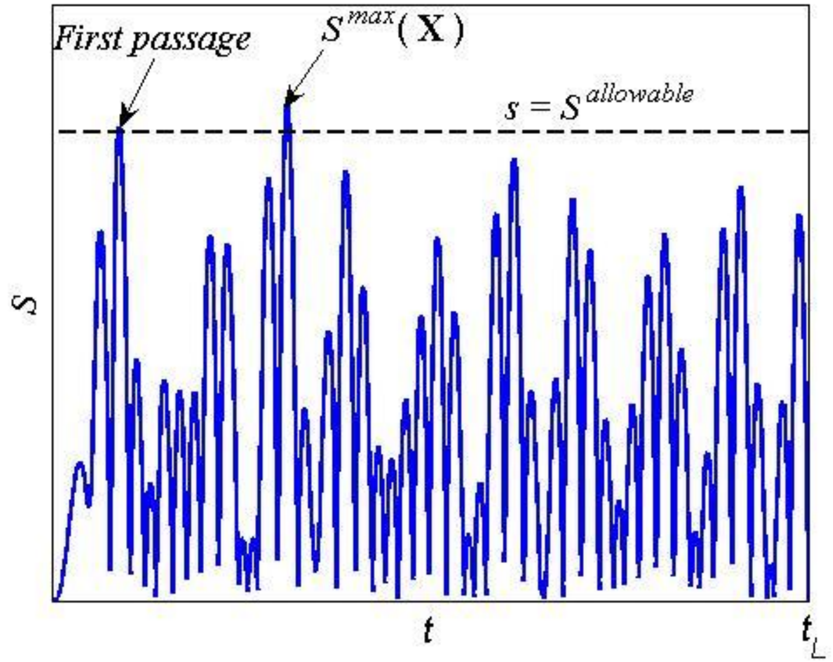
- **Time-dependent reliability**
- **Multiple MPPs**
- **Niching genetic algorithm**

Main Points

- **Time-dependent reliability problem is transformed into a time independent reliability problem for solving a level-crossing problem**

- **Multiple disjoint failure domains may exist**

Time-dependent Reliability Analysis of a Random Process



- **Transfer a time-dependent process to a time independent response:**

$$S = S(\mathbf{X}, t), \quad t_{\min} \le t \le t_{\max} \quad \Rightarrow \quad S^{\max}(\mathbf{X}) = \max_{t_{\min} \le t \le t_{\max}} S(\mathbf{X}, t)$$

- **Time-independent limit state:**

$$g(\mathbf{X}) = S^{\text{allowable}} - S^{\max}(\mathbf{X}) = 0$$

Time-dependent Reliability Analysis of a Random Process

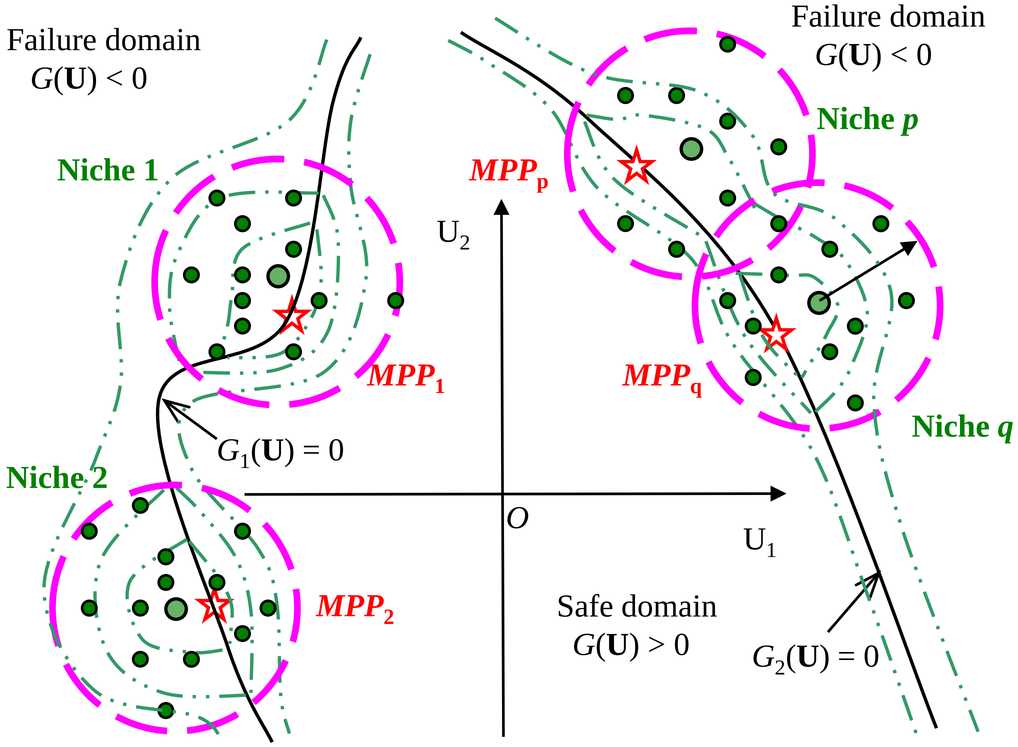
- **Double loop approach:**

$$\begin{array}{l}
\textbf{Outer loop}: \textit{Find MPP's} \\
\quad \beta_j = \min_{\mathbf{U} \in R^n} \|\mathbf{U}\|_2 \\
\quad s.t. \quad g(\mathbf{X}) = S^{allowable}(\mathbf{X}) - S^{\max}(\mathbf{X}) = 0 \\
\qquad \mathbf{X} = \boldsymbol{\mu}_{\mathbf{X}} + \boldsymbol{\sigma}_{\mathbf{X}}\mathbf{U} \\
\qquad \left.\begin{array}{l}
\textbf{Inner loop}: \\
\quad \textit{Find maximum response} \\
\quad \textit{in time domain} \\
\quad S_j^{\max}(\mathbf{X}) = \max_{t_{\min} \le t \le t_{\max}} S_j(\mathbf{X}, t) \\
\textit{End of } \textbf{inner loop}
\end{array}\right\} \textit{Inner loop} \\
\textit{End of } \textbf{outer loop}
\end{array}$$

Outer loop

Time-dependent Reliability Analysis of a Random Process

- **Definitions**
  - ✓ Actual MPPs
  - ✓ Niches
- **Observations**
  - ✓ Niche center is an approximate MPP
  - ✓ Niching GA finds <u>ALL</u> approximate MPPs



Time-dependent Reliability Analysis of a Random Process

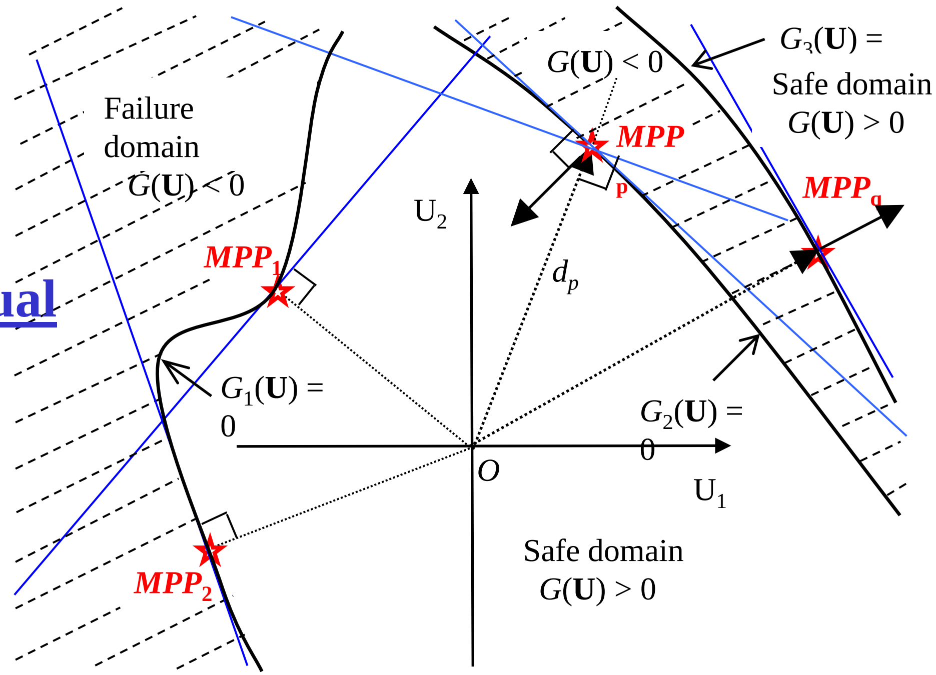
➢ Identification of Actual MPPs

✓ **Linearized limit states**

$$G_p^{\mathrm{L}}(\mathbf{U}) = \left(\mathbf{U} - \mathbf{MPP}_p\right) \cdot \mathbf{MPP}_p = 0$$

or

$$G_p^{\mathrm{L}}(\mathbf{U}) = \left(\mathbf{U} - \mathbf{MPP}_p\right) \nabla G_p = 0$$



✓ **Collinearity criterion for <u>actual</u> MPP**

$$\cos\angle\left(\mathbf{MPP}_p, \nabla G_p\right) = \frac{\mathbf{MPP}_p}{\left\|\mathbf{MPP}_p\right\|} \cdot \frac{\nabla G_p}{\left\|\nabla G_p\right\|} = \pm 1$$

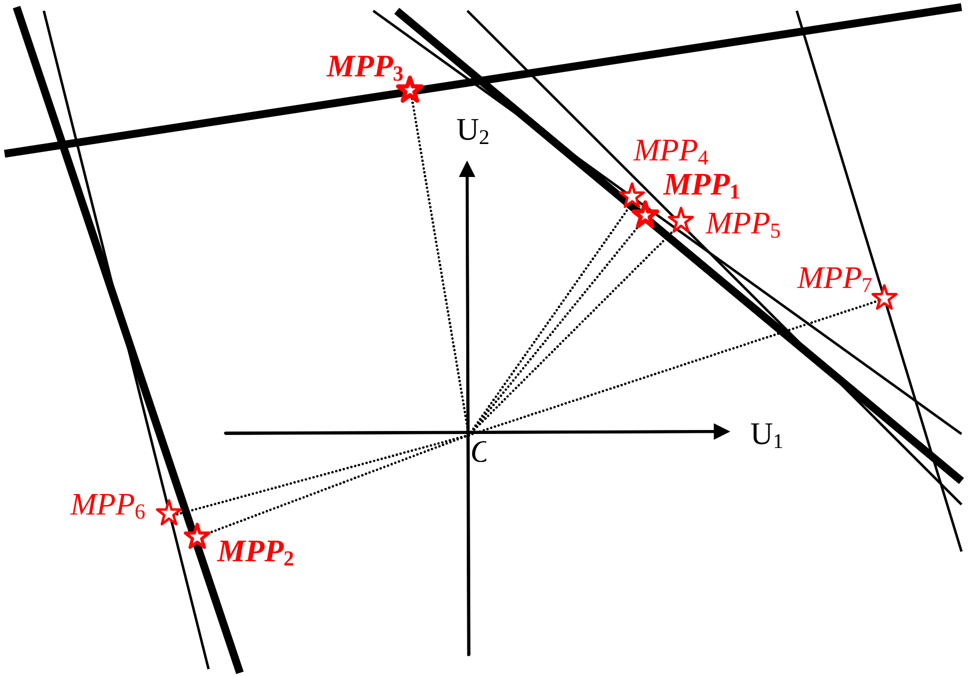
✓ **Practical criterion**

$$\left| \frac{\mathbf{MPP}_p}{\left\|\mathbf{MPP}_p\right\|} \cdot \frac{\nabla G_p}{\left\|\nabla G_p\right\|} \right| \geq 1 - \varepsilon_{\mathrm{MPP}} \qquad 0 < \varepsilon_{\mathrm{MPP}} < 1$$

Time-dependent Reliability Analysis of a Random Process

- **Identification of Independent and Significant MPPs**
  - **Two MPPs are <u>independent</u> if the correlation coefficient is very close to +1**
  - **Reliability index identifies <u>significant</u> MPPs**

$$\rho_{pq}^{\mathrm{L}} = \cos\angle\left(\mathbf{MPP}_p, \mathbf{MPP}_q\right) = \frac{\mathbf{MPP}_p}{\left\|\mathbf{MPP}_p\right\|} \cdot \frac{\mathbf{MPP}_q}{\left\|\mathbf{MPP}_q\right\|}$$



Time-dependent Reliability Analysis of a Random Process

Estimation of Probability of Failure

- Convex safe domain formed by linearized limit states

$$M_p: \quad G_p^{\mathrm{L}}(\mathbf{U}) = \mathbf{U}\cdot\mathbf{MPP}_p - \mathbf{MPP}_p\cdot\mathbf{MPP}_p \le 0 \quad \text{for } p = 1,2,\ldots,M \qquad \beta_p^{\mathrm{L}} = \left\|\mathbf{MPP}_p\right\|_2$$

- Second order (bi-modal) bounds

$$p_{f_1} + \sum_{p=2}^{M}\max\left(p_p - \sum_{q=1}^{p-1}P\left(M_p \cap M_q\right), 0\right) \le p_f \le \sum_{p=1}^{M}p_{f_i} - \sum_{p=2}^{M}\max_{q<p}P\left(M_p \cap M_q\right)$$

- MCS on the linearized convex safe domain

$$\Omega_{\mathrm{f}}^{\mathrm{L}} = \bigcup_{p}^{M} M_p = \bigcup_{p}^{M}\left(\mathbf{U}\cdot\mathbf{MPP}_p - \mathbf{MPP}_p\cdot\mathbf{MPP}_p \le 0\right)$$

A Niching GA Method for Identifying Multiple MPPs

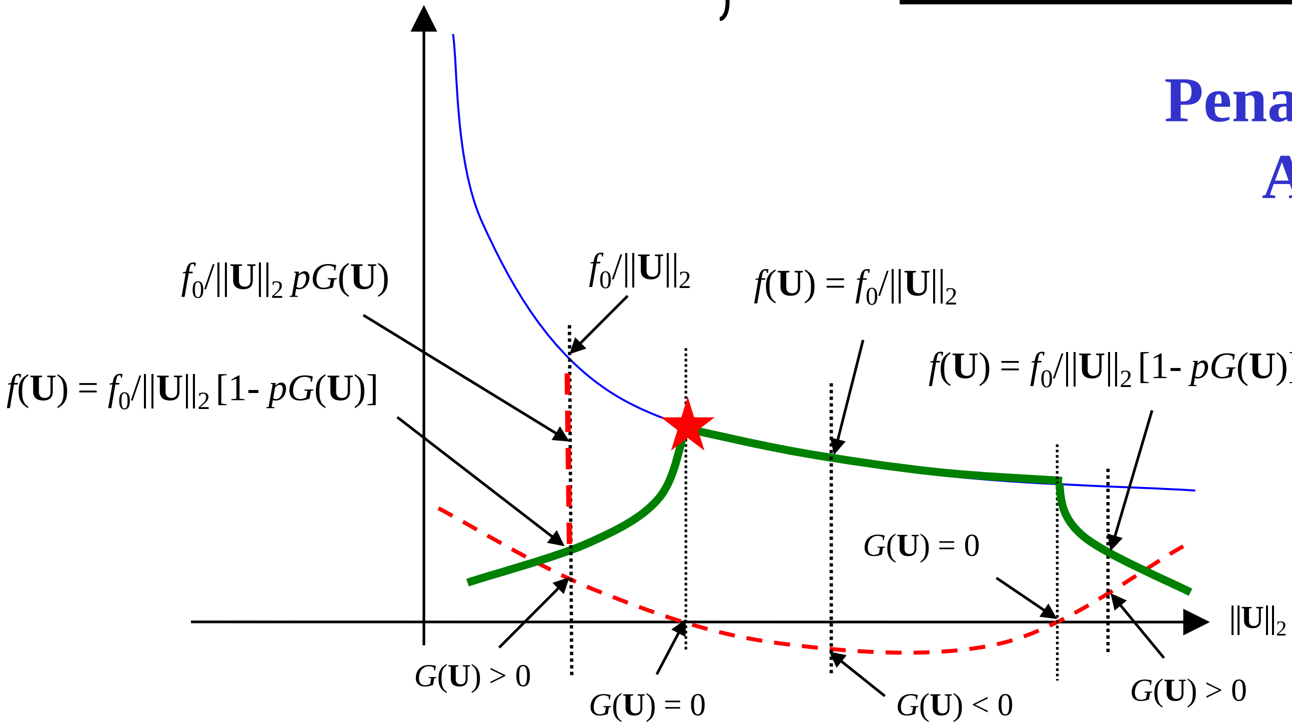
- **Greedy Fitness Sharing**
  - **Fitness sharing is a popular niching technique.**
    - **Decreases chance of mating in densely populated niches**
    - **Increases chance of mating in sparse niches.**

A Niching GA Method for Identifying Multiple MPPs

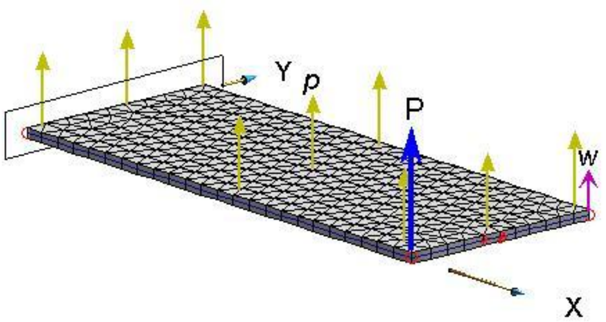
- ✓ **Fitness function for identification of MPPs using RIA**

$$\left.\begin{aligned} \beta_j &= \min_{\mathbf{U}} \left\|\mathbf{U}\right\|_2 \\ s.t.\quad g(\mathbf{U}) &= 0 \end{aligned}\right\} \Rightarrow \boxed{\max\ f(\mathbf{U}) = \frac{f_0}{\left\|\mathbf{U}\right\|_2}\{1 - p\max[G(\mathbf{U}), 0]\}}$$

Penalty Function Approach



Numerical Example



•Model Description

- ➢3% critical modal damping
- ➢Carried out for 0.1 seconds.

$$P(t) = 25\cos(300\pi\, u(t - 0.004)) \quad lbf$$

$$p(t) = \cos(500\pi t) \quad psi$$

$$w(t) < \underbrace{0.3 \quad in}_{|w|_{\max}}$$

Numerical Example

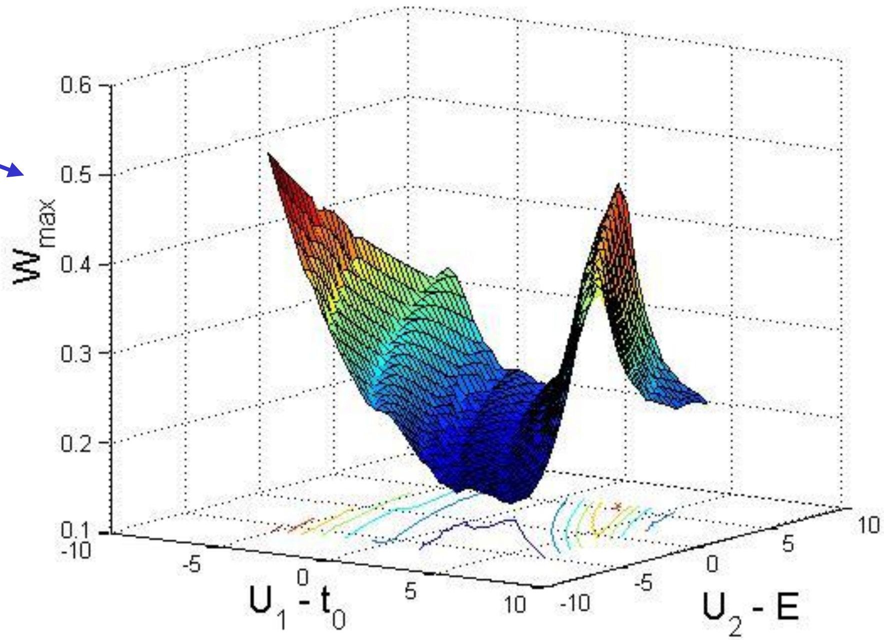
Plate properties

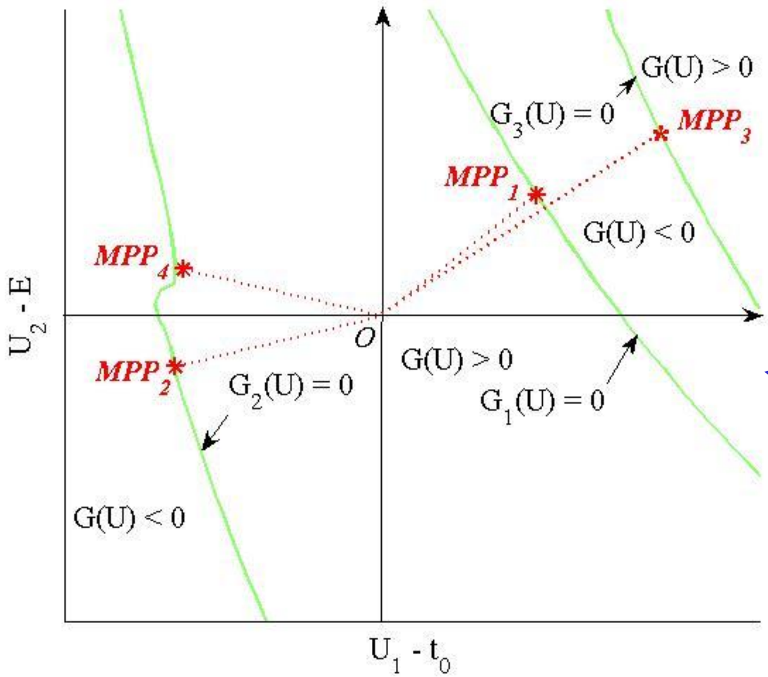
	Mean μ_X	$Cov = \frac{\sigma_X}{\mu_X}$	Std. Variation σ_X
Length (a)	2in	0	0
Height (b)	2in	0	0
Thickness	**0.0877in**	**0.0513**	**0.0045in**
Weight Density	0.282 lbs/in^3	0	0
Mass/Weight Factor	2.59×10^{-3} sec^2/in	0	0
Young's Modulus	**30.0×10^6 lbs/in^2**	**0.05**	**15×10^5 lbs/in^2**
Poisson's Ratio	0.3	0	0

Numerical Example

Multimodal maximum displacement response

$$S^{\max} = w_{\max}$$





Limit State :

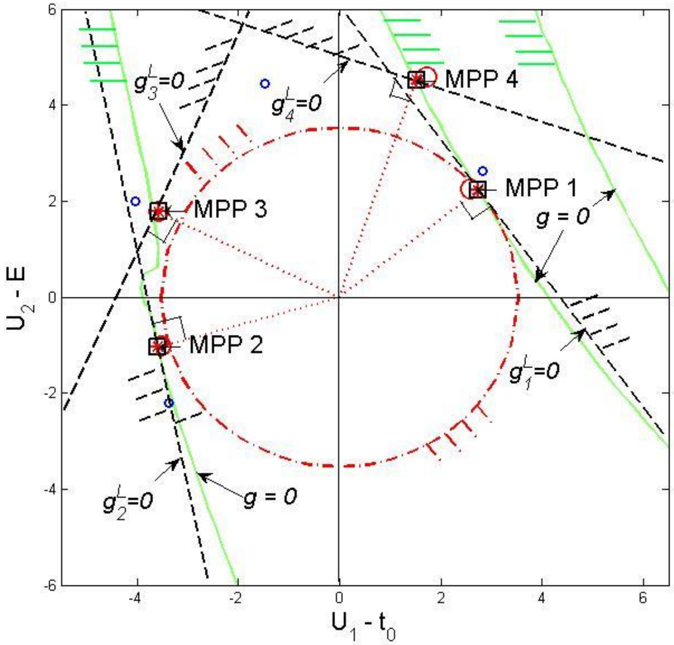
$$G(\mathbf{U}) = w_{\max} - 0.3 = 0$$

Numerical Example

- **Identification of independent and significant MPPs.**
  - **Form a <u>linear approximation</u> of each limit state at each MPP.**

$$\rho^{L} = \left[\rho^{L}_{pq}\right] = \begin{bmatrix} 1 & -0.9213 & -0.4146 & 0.8444 \\ -0.9213 & 1 & 0.7359 & -0.5696 \\ -0.4146 & 0.7359 & 1 & 0.1373 \\ 0.8444 & -0.5696 & 0.1373 & 1 \end{bmatrix}$$

Independent MPPs: $\rho_{ij} < +0.9$



Numerical Example

- **Estimation of probability of failure using the Ditlevson's second-order bounds:**

$$p_f^L = p_f^U = 3.3298 \times 10^{-4}$$

MC simulation with 1,000,000 samples using linearized safe domain yields:

$$P_{\mathrm{f\,MCS}}^{\mathrm{L}} = 3.51 \times 10^{-4}$$

If MPP4 is removed: $p_f = 3.3287 \times 10^{-4}$ ⟹

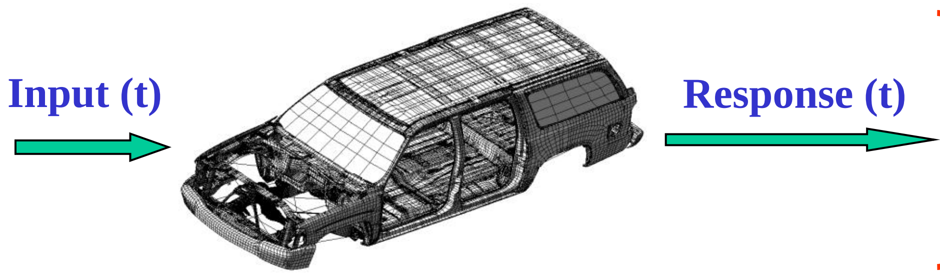
⟹ **MPP4 is insignificant**

Observations / Future Work

- Calculation of **cumulative prob. of failure**

  ➢ Series system approach for $[0, t_f]$

  ➢ Reliability estimation at time t_L ($0 < t_L < t_f$) ; **multiple MPP case**



- **Deterministic** Re-Analysis method using PROM, MCA and Kriging
- Time-dependent metamodeling

- Series system probabilistic analysis

  - **Probabilistic** Re-Analysis method using single MC simulation

- Efficient solution of design optimization problem for lifecycle cost

Q & A

Disclaimer: Reference herein to any specific commercial company, product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the Department of the Army (DoA). The opinions of the authors expressed herein do not necessarily state or reflect those of the United States Government or the DoA, and shall not be used for advertising or product endorsement purposes.